Particle production in field theories coupled to a strong source

François Gelis

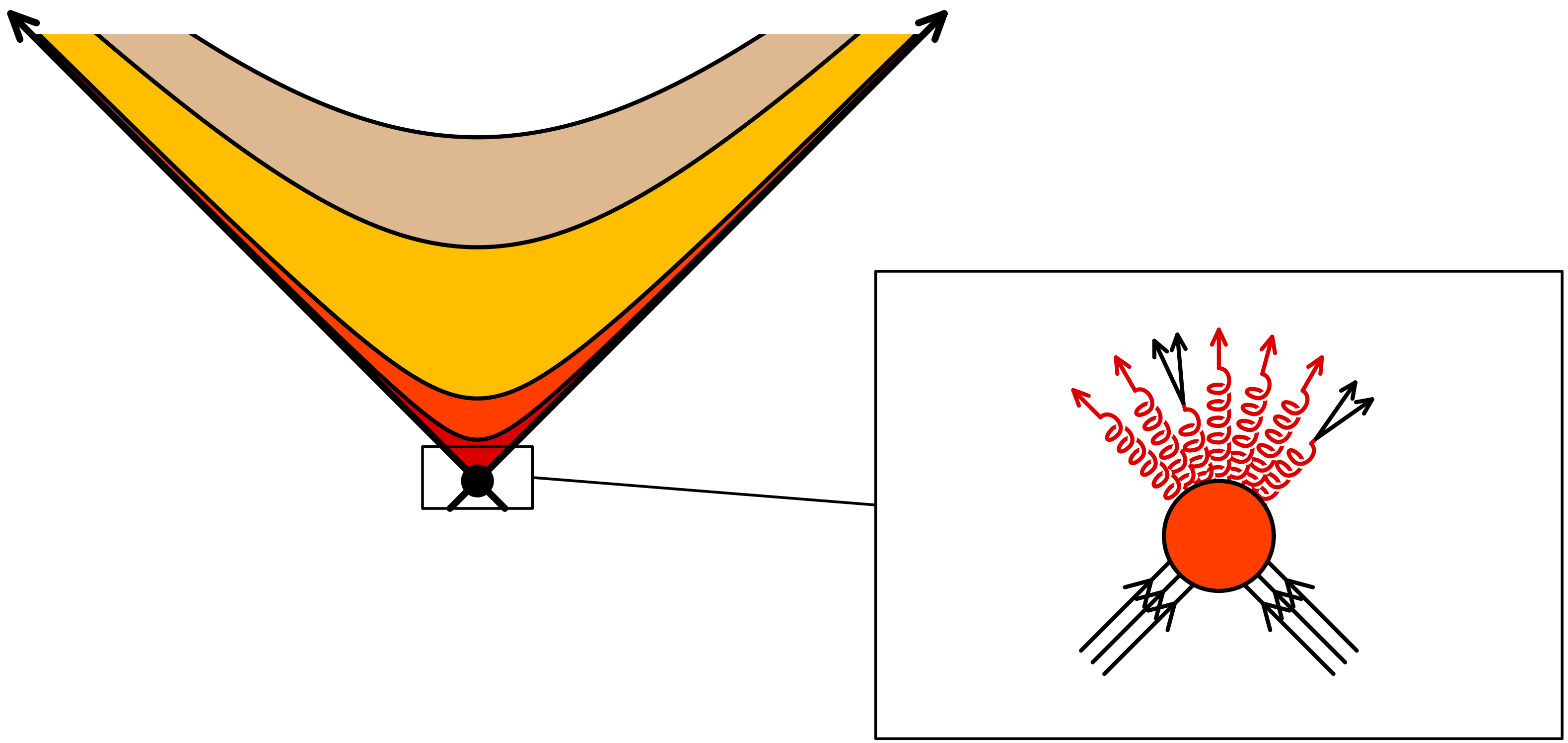
CEA / DSM / SPhT

Outline

- High energy hadrons and Color Glass Condensate
- General properties of field theories coupled to an external source
- First moment – Average multiplicity at LO and NLO
- Generating function for the particle multiplicities
- Towards kinetic theory

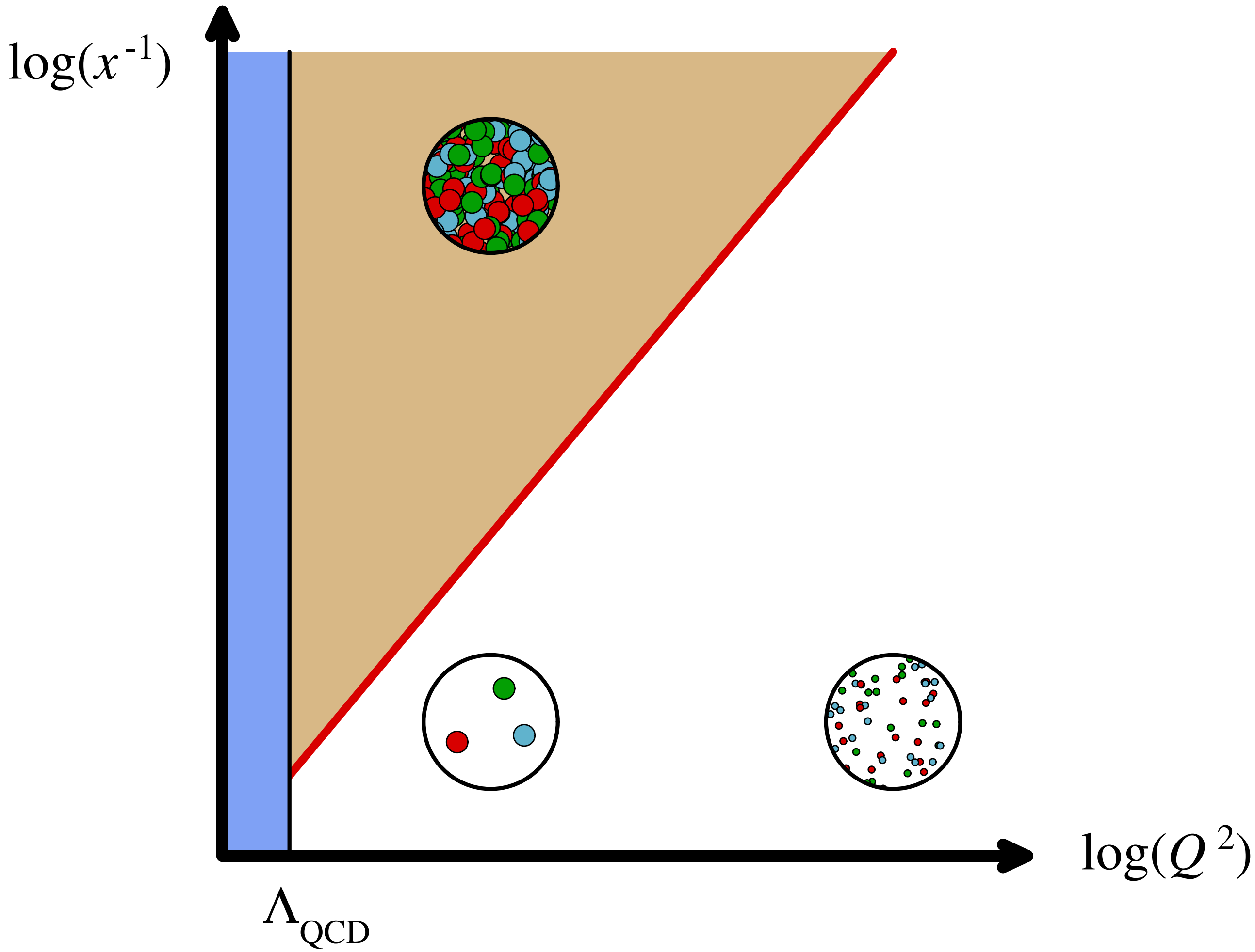
  - Baltz, FG, McLerran, Peshier, nucl-th/0101024
  - FG, Kajantie, Lappi, hep-ph/0409058, 0508229
  - FG, Venugopalan, hep-ph/0601209

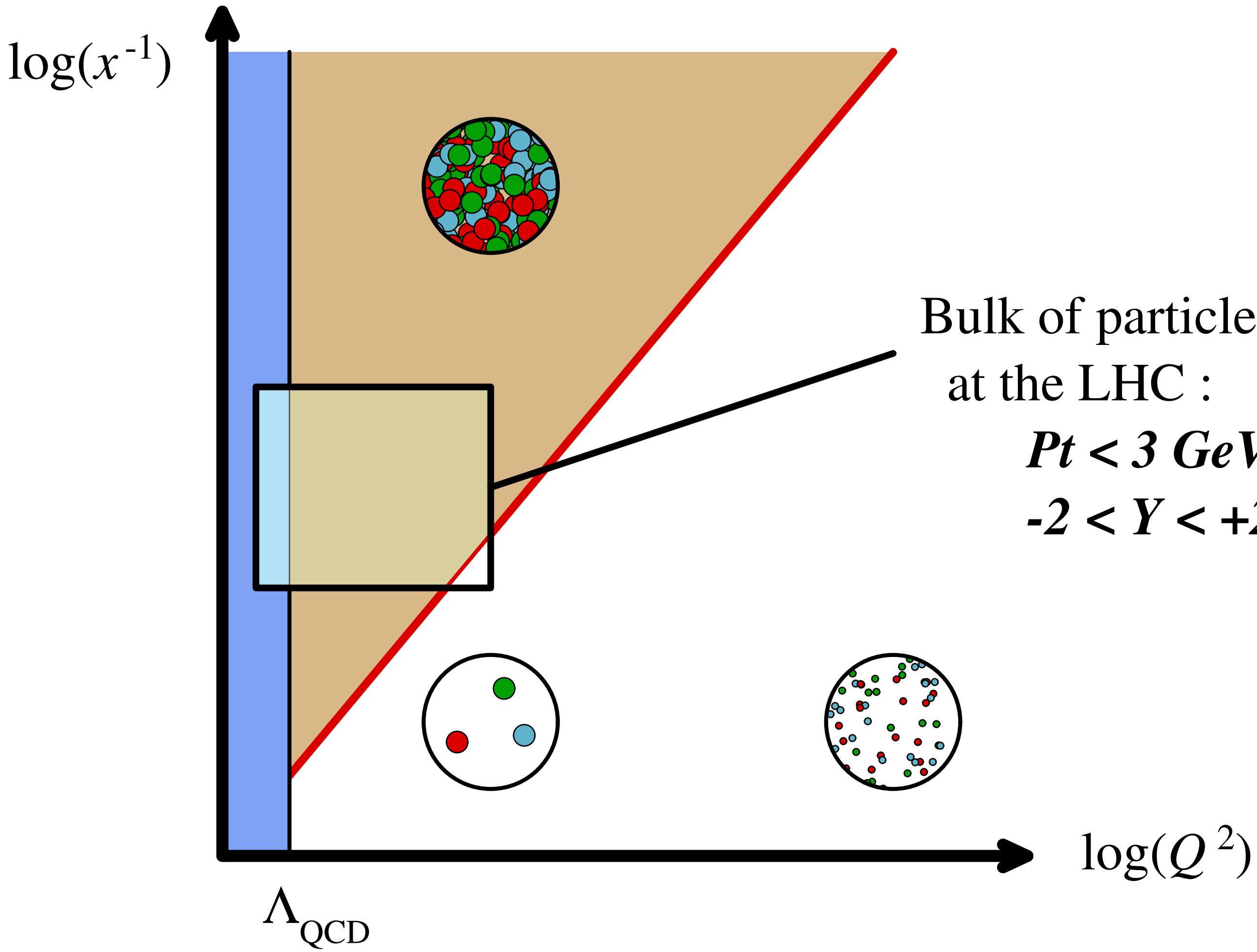
Goals



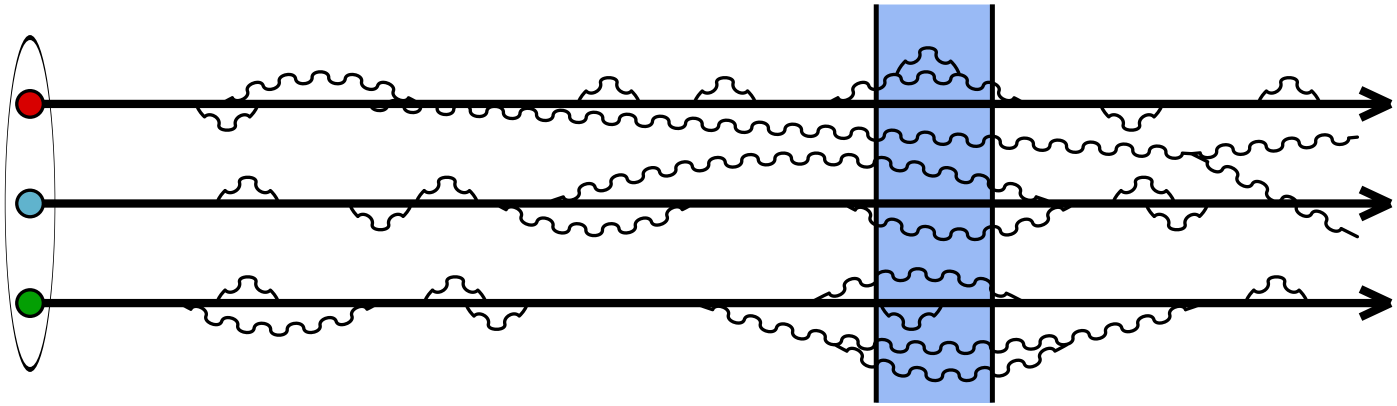
- describe the semi-hard content of nucleons and nuclei
- calculate the initial production of semi-hard particles in high-energy heavy ion collisions

Saturation domain




$\log(x^{-1})$
Bulk of particle production
at the LHC :
Pt < 3 GeV
-2 < Y < +2
$\log(Q^2)$
Λ_{QCD}

Nucleon at high energy



- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  - ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. The nucleon appears denser at high energy (it contains more gluons)
- Pre-existing fluctuations are frozen over the time-scale of the probe, and act as static sources of new partons

Degrees of freedom and their interplay

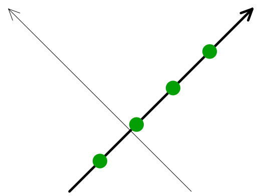
McLerran, Venugopalan (1994), Iancu, Leonidov, McLerran (2001)

- Small-x modes have a large occupation number
  ▷ they are described by a classical color field A^μ, that obeys Yang-Mills's equation:

$$[D_\nu, F^{\nu\mu}] = J^\mu$$

- The source term J^μ comes from the faster partons. The large-x modes, slowed down by time dilation, are described as frozen color sources ρ. Hence :

$$J^\mu = \delta^{\mu+}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp)$$

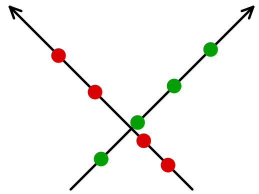


- The color sources ρ are random, and described by a distribution functional $W_{x_0}[\rho]$, with x_0 the frontier between "small-x" and "large-x"

Calculation of observables

- In order to study the collisions of two hadrons, solve the classical Yang-Mills equations in the presence of the following current :

$$J^\mu \equiv \delta^{\mu+}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$



- Compute the observable $\mathcal{O}$ of interest in the background field created by a configuration of the sources ρ_1, ρ_2. Note : the sources are of order $1/g$ ▷ this is a very non-linear problem

- Average over the sources ρ_1, ρ_2

$$\langle\mathcal{O}\rangle = \int [D\rho_1]\,[D\rho_2]\;W_{x_1}[\rho_1]\;W_{x_2}[\rho_2]\;\mathcal{O}[\rho_1,\rho_2]$$

- Note: in the rest of this talk, I'll assume that the distributions $W_x[\rho]$ of sources are known

Toy model

- From now on, we assume that $j = j_1 + j_2$, with j_1 and j_2 of comparable strengths
- The sources can be as strong as $1/g$ in the saturated regime: ▷ corrections in $(gj)^n$ must be summed to all orders, which makes the evaluation of physical quantities very complicated – even at “leading order”
- To avoid encumbering the discussion with unessential details, consider a scalar field theory with a ϕ^3 coupling, coupled to a source $j(x)$:

$$\mathcal{L} \equiv \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3 + j\phi$$

Counting the powers of g

- Consider a diagram with :
  - E external lines
  - I internal lines
  - V vertices
  - J sources
  - L independent loops
- These numbers are related by :

$$3V + J = E + 2I$$

$$L = I - J + 1$$

- Therefore, the order of the diagram in g and j is :

$$g^V j^J = g^{E+2(L-1)} (gj)^J$$

- After resummation of all the powers of gj, the order of a diagram depends only on its number of loops and external legs

Reduction formulas

- Production of a single particle :

$$\big\langle \vec{\boldsymbol{p}}_{\rm out}\big|0_{\rm in}\big\rangle = \frac{1}{Z^{1/2}}\int d^4x\; e^{ip\cdot x}\;(\square_x+m^2)\big\langle 0_{\rm out}\big|\phi(x)\big|0_{\rm in}\big\rangle$$

- Production of a two particles :

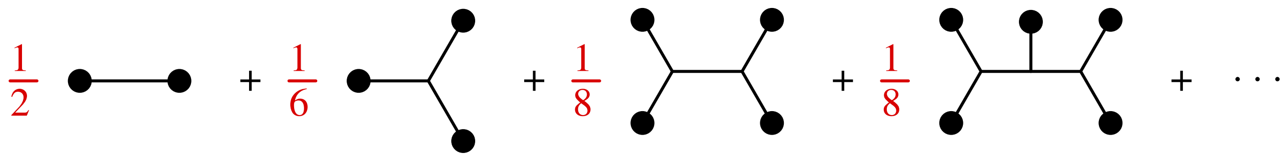
$$\begin{aligned}\big\langle \vec{\boldsymbol{p}}\vec{\boldsymbol{q}}_{\rm out}\big|0_{\rm in}\big\rangle &= \frac{1}{Z}\int d^4x\,d^4y\; e^{iq\cdot y}e^{ip\cdot x}\\ &\qquad\times(\square_x+m^2)(\square_y+m^2)\big\langle 0_{\rm out}\big|T\phi(x)\phi(y)\big|0_{\rm in}\big\rangle\end{aligned}$$

Vacuum-vacuum diagrams

- The perturbative expansion of these transition amplitude generates vacuum-vacuum diagrams (i.e. disconnected diagrams that do not have any external leg).
  Their sum is not a pure phase
- The sum of all the vacuum-vacuum diagrams in $\big<0_{\rm out}\big|0_{\rm in}\big>$ is the exponential of the sum of the connected ones

$$\big<0_{\rm out}\big|0_{\rm in}\big> = e^{iV[j]}$$

- The perturbative expansion of $iV[j]$ starts with :



  Note : each graph Γ comes with a symmetry factor $1/S_\Gamma$
- Note : $\exp(iV[j])$ can be seen as a generating functional :

$$\big<0_{\rm out}\big|T\phi(x_1)\cdots\phi(x_n)\big|0_{\rm in}\big> = \frac{\delta}{i\delta j(x_1)}\cdots\frac{\delta}{i\delta j(x_n)}\;\; e^{iV[j]}$$

Generating function

- The probability of producing exactly n particles is :

$$P_n = \frac{1}{n!}\int \frac{d^3\vec{\boldsymbol{p}}_1}{(2\pi)^3 2E_1}\cdots\frac{d^3\vec{\boldsymbol{p}}_n}{(2\pi)^3 2E_n}\left|\left\langle \vec{\boldsymbol{p}}_1\cdots\vec{\boldsymbol{p}}_{n\,\mathrm{out}}\middle|0_{\mathrm{in}}\right\rangle\right|^2$$

- Generating function : $F(z) \equiv \sum_{n=0}^{+\infty} P_n\, z^n$

- One can show that :

$$F(z) = e^{z\mathcal{D}[j_+,j_-]}\; e^{iV[j_+]}\; e^{-iV^*[j_-]}\Big|_{j_+=j_-=j}$$

with

$$\mathcal{D}[j_+,j_-] \equiv \frac{1}{Z}\int_{x,y} G^0_{+-}(x,y)\,(\square_x+m^2)(\square_y+m^2)\,\frac{\delta}{\delta j_+(x)}\frac{\delta}{\delta j_-(y)}$$

$$G^0_{+-}(x,y) \equiv \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_p}\, e^{ip\cdot(x-y)}$$

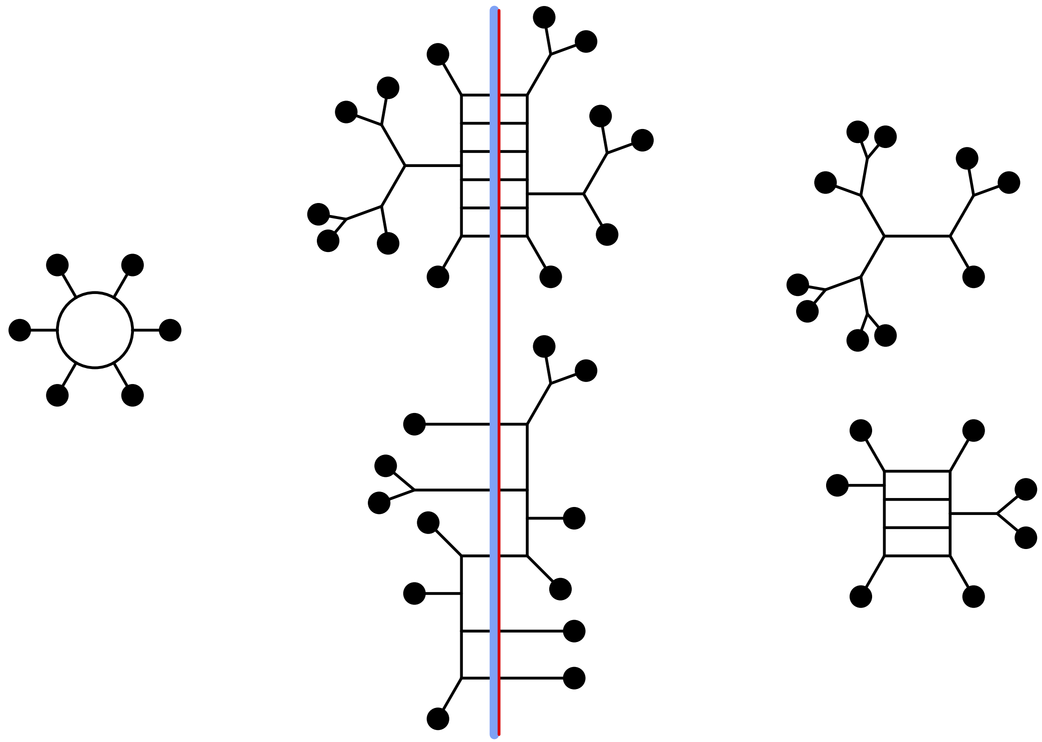
Interpretation of F(z)

- $\exp(iV[j_+])$ is obtained with the usual Feynman rules :
  - Propagator : $G^0_{++}(p) = i/(p^2 - m^2 + i\epsilon)$
- $\exp(-iV^*[j_-])$, is obtained with the conjugate rules :
  - Propagator : $G^0_{--}(p) = -i/(p^2 - m^2 - i\epsilon)$
- Schwinger-Keldysh formalism :
  - For each graph, assign a sign ϵ to each vertex or source, in all the possible ways $\triangleright$ 2^{V+J} terms
  - If the sign is $+$: vertex $-ig$ and source $+ij_+$
  - If the sign is $-$: vertex $+ig$ and source $-ij_-$
  - Connect the vertices ϵ and ϵ' with the propagator $G^0_{\epsilon\epsilon'}$
- The action of $\exp(\mathcal{D}[j_+, j_-])$ is to build the mixed diagrams
- The generating function has the following interpretation :

$F(z)$ is the sum of all the Schwinger-Keldysh vacuum-vacuum diagrams, in which each propagator of type $+-$ or $-+$ is weighted by a factor z

Why calculating Pn is hard

- Example : typical contribution to P_{11} :



- At tree level, all the disconnected graphs are of order $1/g^2$
  ▷ therefore, no truncation is possible for the leading order

Calculation of the moments

- Start from :

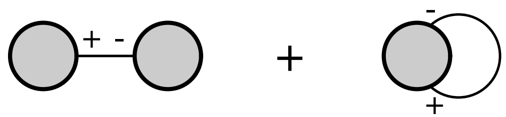
$$F(z)=\sum_n P_n z^n = e^{z\mathcal{D}[j_+,j_-]}\;\, e^{iV[j_+]}\;\, e^{-iV^*[j_-]}\Big|_{j_+=j_-=j}$$

- The average multiplicity is given by :

$$\langle n\rangle \quad = \quad F'(1)=\mathcal{D}[j_+,j_-]\;\underbrace{e^{\mathcal{D}[j_+,j_-]}e^{iV[j_+]}e^{-iV^*[j_-]}}_{e^{iW[j_+,j_-]}}\Big|_{j_+=j_-=j}$$

- More explicitly, this reads :

$$\langle n\rangle = \frac{1}{Z}\int_{x,y} G^0_{+-}(x,y)\;(\square_x+m^2)(\square_y+m^2)\left[\underbrace{\frac{\delta iW}{\delta j_+(x)}\,\frac{\delta iW}{\delta j_-(y)}}_{}+\underbrace{\frac{\delta^2 iW}{\delta j_+(x)\delta j_-(y)}}_{}\right]$$

Or, diagrammatically : $\langle n\rangle \quad =$ 

Leading Order

- At leading order – i.e. $\mathcal{O}(1/g^2)$ – we need only tree diagrams :

- For all the vertices except the two which are labelled explicitly, we must sum over the indices $+/-$
- We must also sum over all the topologies for the tree diagrams on the left and on the right of the G^0_{+-} propagator
- By using repeatedly the relation $G^0_{++} - G^0_{+-} = G^0_{-+} - G^0_{--} = G_R$, the only effect of the summation over the $+/-$ indices is to turn all the propagators into retarded propagators

Leading Order

- The sum of all 1-point tree diagrams made of retarded propagators is the solution ϕ_R of the classical equation of motion

$$(\square_x + m^2)\phi_R(x) + \frac{g}{2}\phi_R^2(x) = j(x)$$

with a null retarded boundary condition

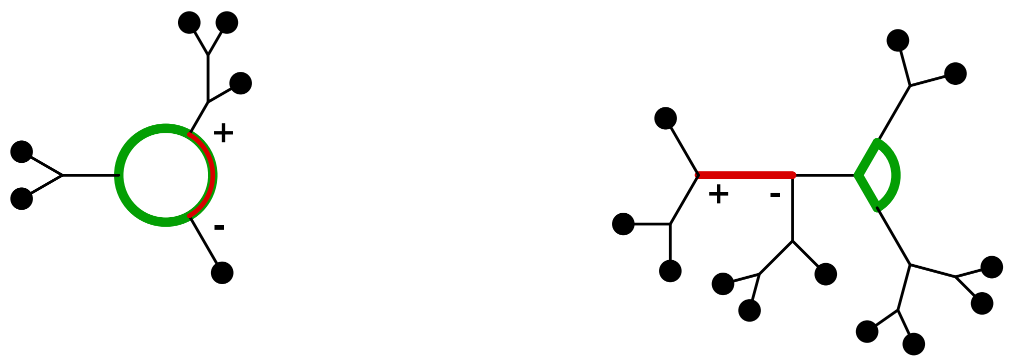
$$\lim_{x^0\to-\infty} \phi_R(x) = 0 \; , \qquad \lim_{x^0\to-\infty} \partial^0\phi_R(x) = 0$$

- Finally, one obtains :

$$E_p \left.\frac{d\langle n\rangle}{d^3\vec{\boldsymbol{p}}}\right|_{LO} = \frac{1}{16\pi^3}\int_{x,y} e^{ip\cdot(x-y)}(\square_x + m^2)(\square_y + m^2)\,\phi_R(x)\phi_R(y)$$

Next to Leading Order

- There are two types of corrections at NLO :



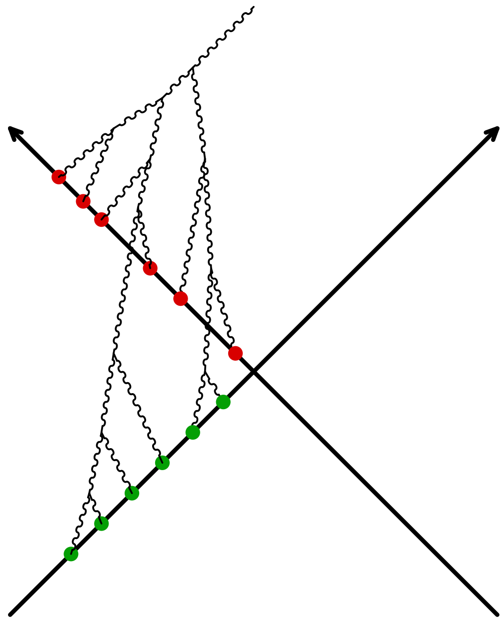
- They both contribute at order g^0. For quark production, the first type of NLO topologies would in fact be the leading contribution
- **One can show that, at NLO, the summation of all the diagrams involved in $\langle n \rangle$ can be performed by solving the EOM for small fluctuations on top of the classical field**

Gluon production

Krasnitz, Nara, Venugopalan (1999, 2001), Lappi (2003)

$$E_{\boldsymbol{p}} \frac{d\langle n_{\text{gluons}}\rangle}{d^3\vec{\boldsymbol{p}}} = \frac{1}{16\pi^3} \int_{x,y} e^{ip\cdot(x-y)}\, \square_x \square_y\, \langle A(x)A(y)\rangle$$

- At LO, one just needs to solve Yang-Mills equations, with retarded boundary conditions :



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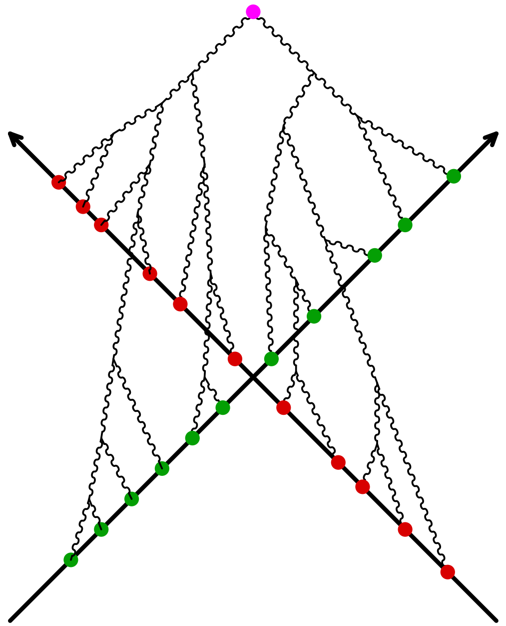
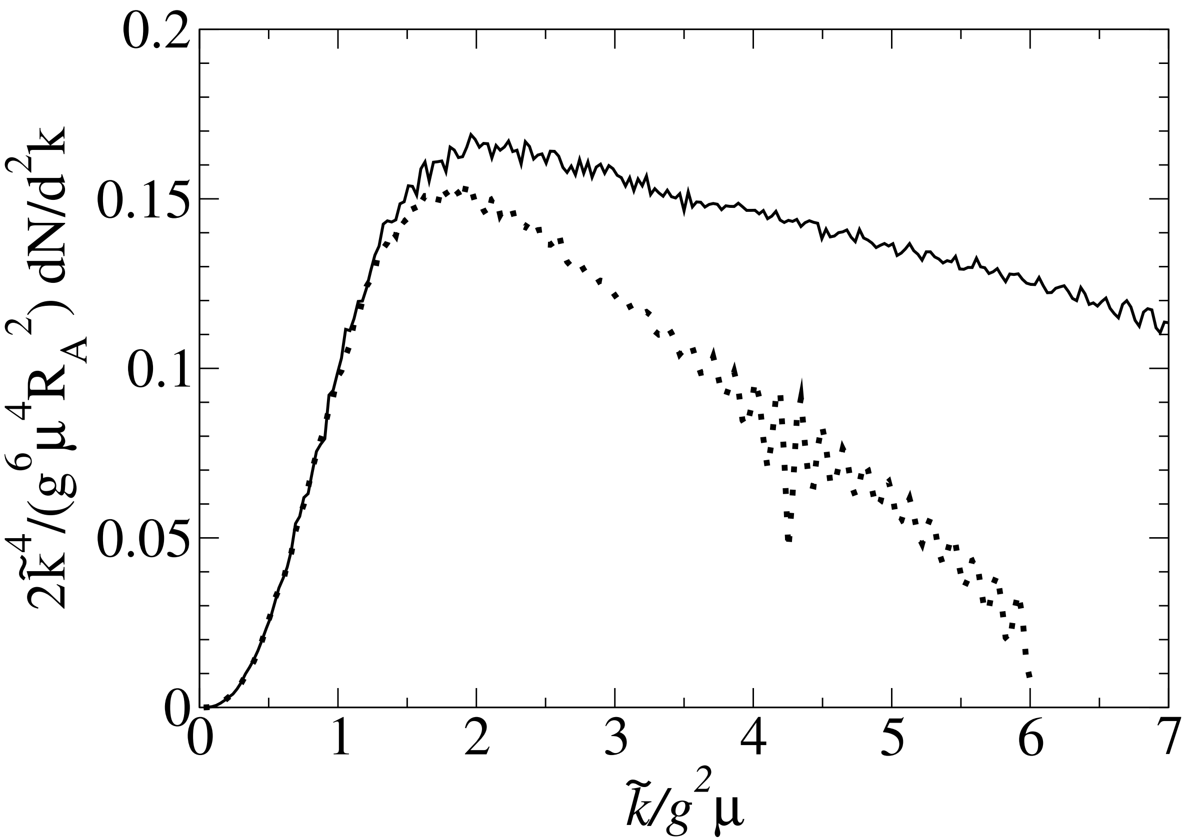


Gluon spectrum

- Gluon spectra on 256^2 and 512^2 transverse lattices:



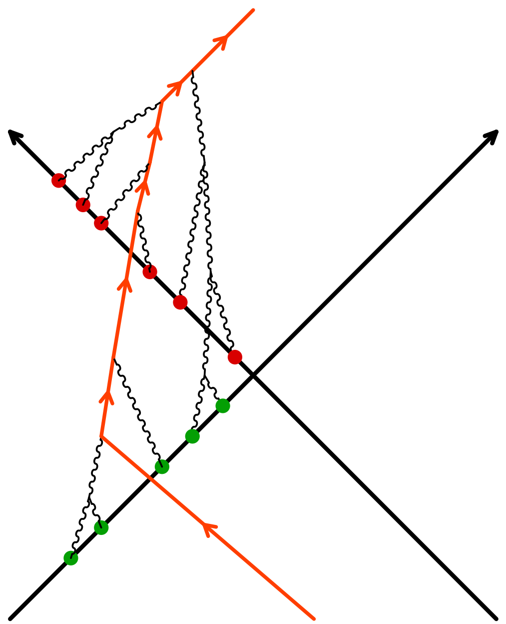
  - Lattice cutoff at large momentum (they do not affect much the overall number of gluons)
  - Important softening at small $k_\perp$ compared to pQCD (saturation)

Quark production

FG, Kajantie, Lappi (2004, 2005)

$$E_{\boldsymbol{p}} \frac{d\langle n_{\text{quarks}} \rangle}{d^3\vec{\boldsymbol{p}}} = \frac{1}{16\pi^3} \int_{x,y} e^{ip\cdot(x-y)}\, \not{\partial}_x \not{\partial}_y \, \langle \overline{\psi}(x)\psi(y) \rangle$$

- Dirac equation in the classical color field :



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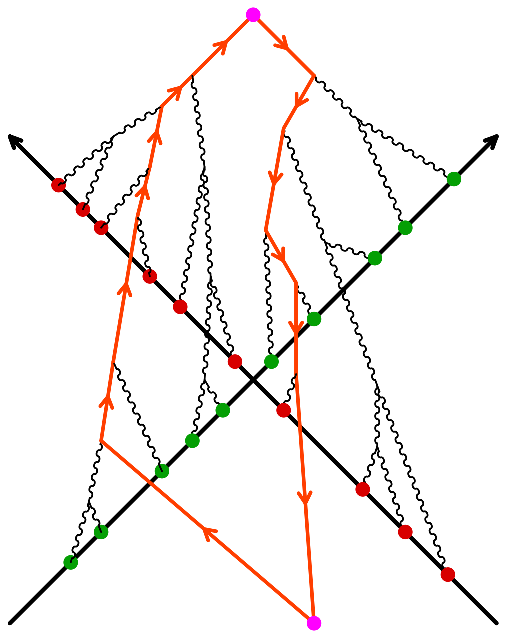
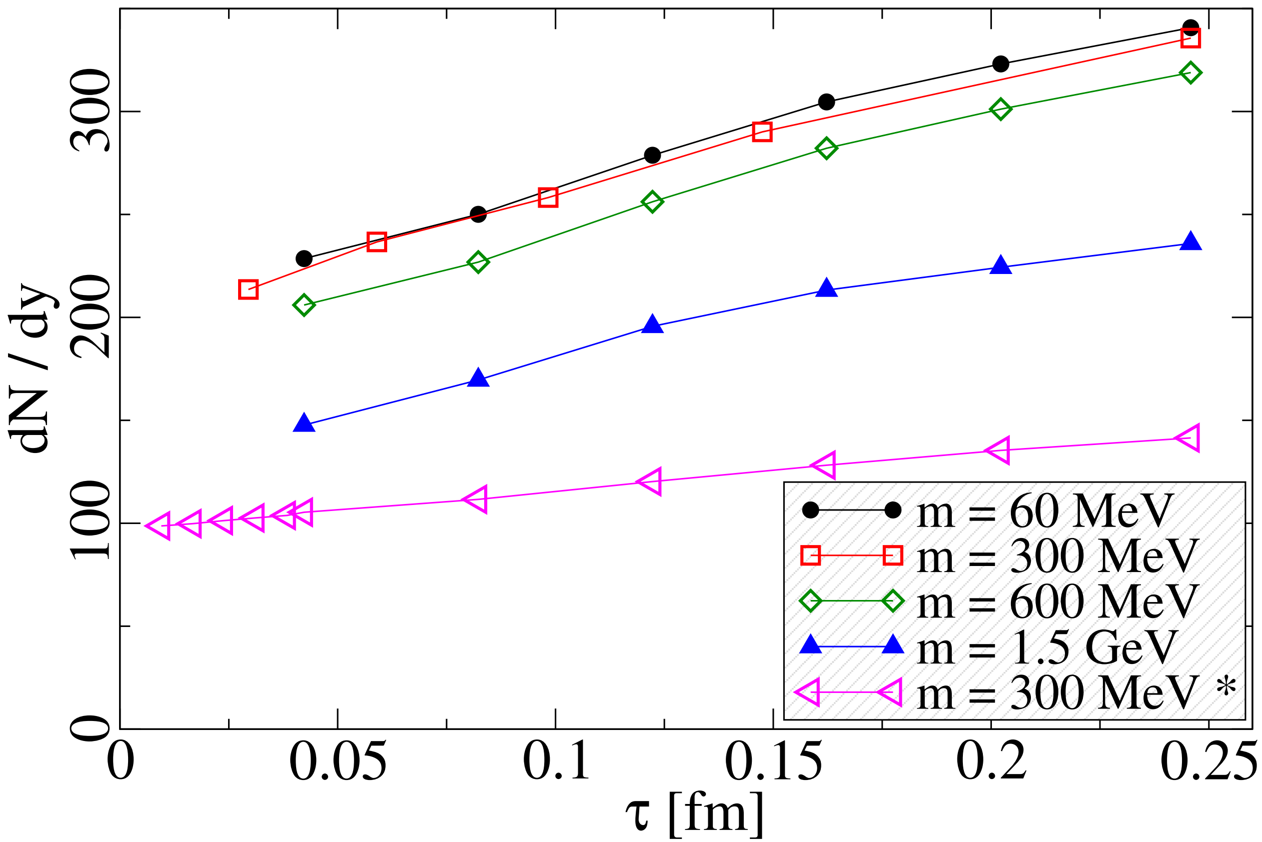
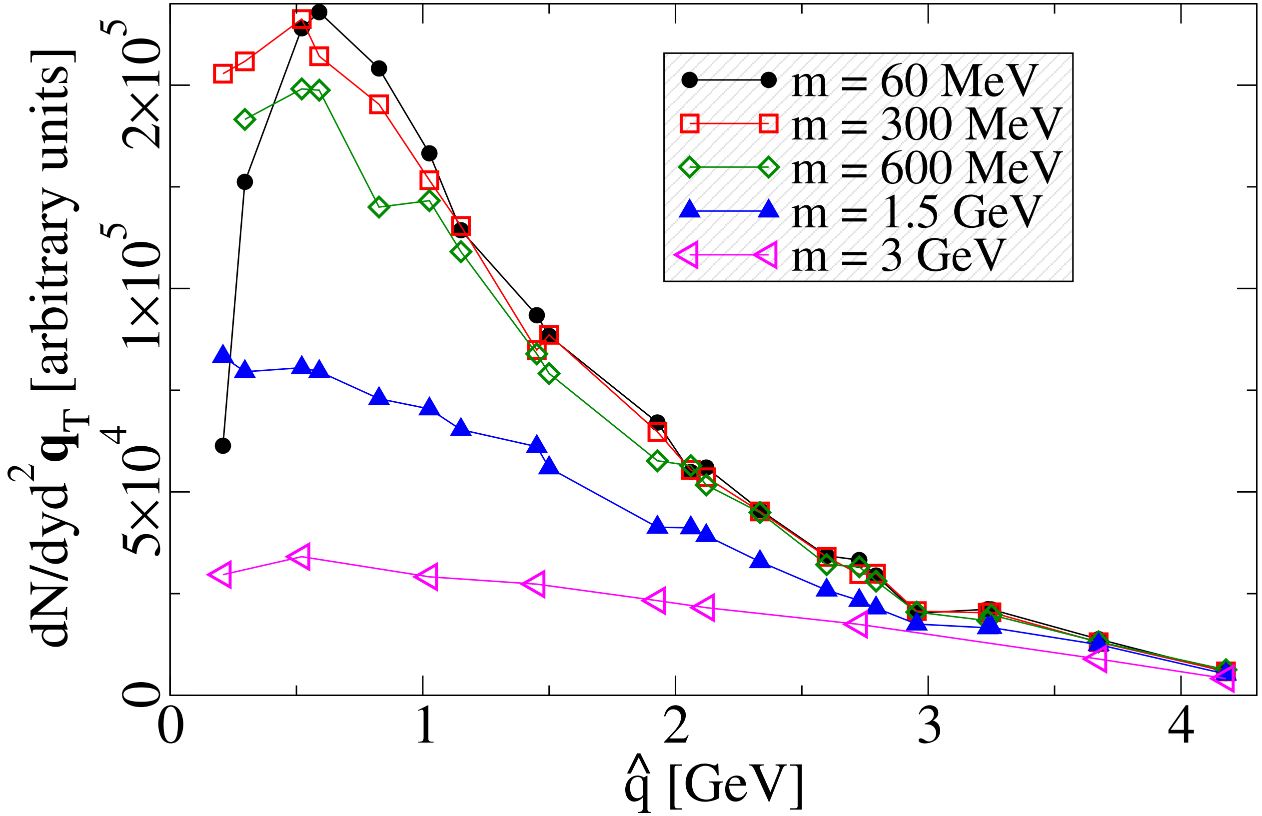


Time dependence of quark production



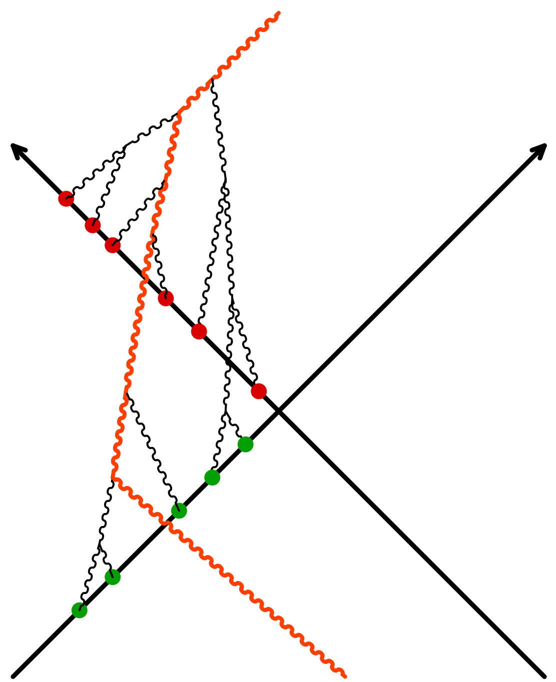
Spectra for various quark masses



Gluon production at NLO (1/2)

FG, Lappi, Venugopalan (work in progress)

- A part of the NLO correction is very similar to quark production : it involves the EOM for small field fluctuations on top of the classical solution



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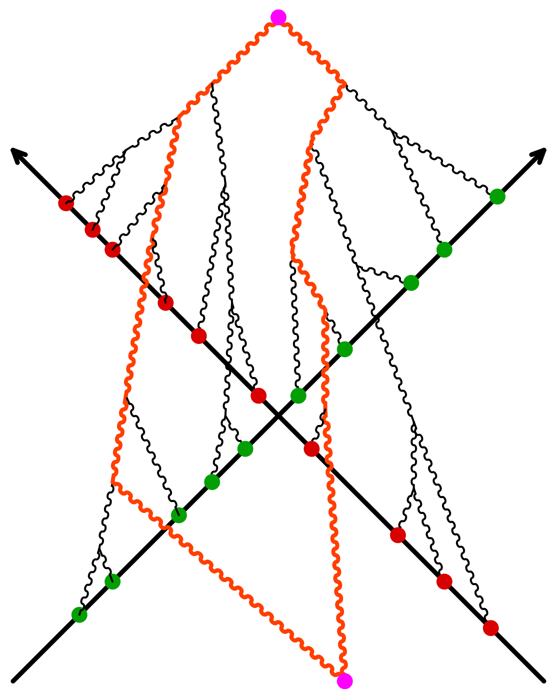
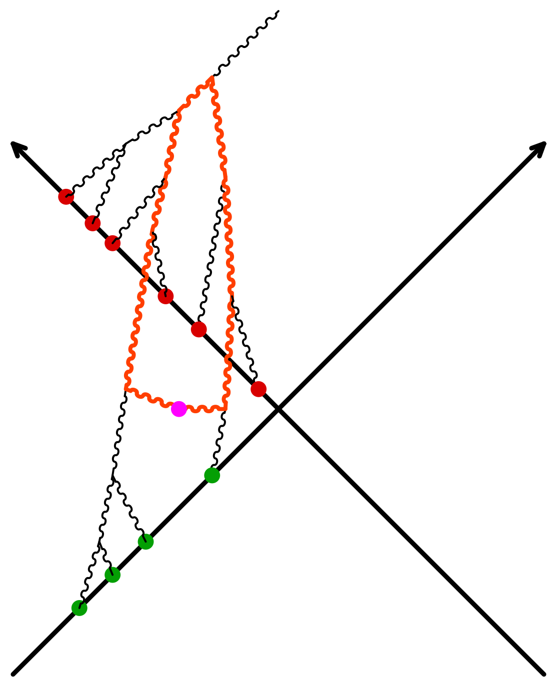


Gluon production at NLO (2/2)

- The other part is a 1-loop correction to the classical field



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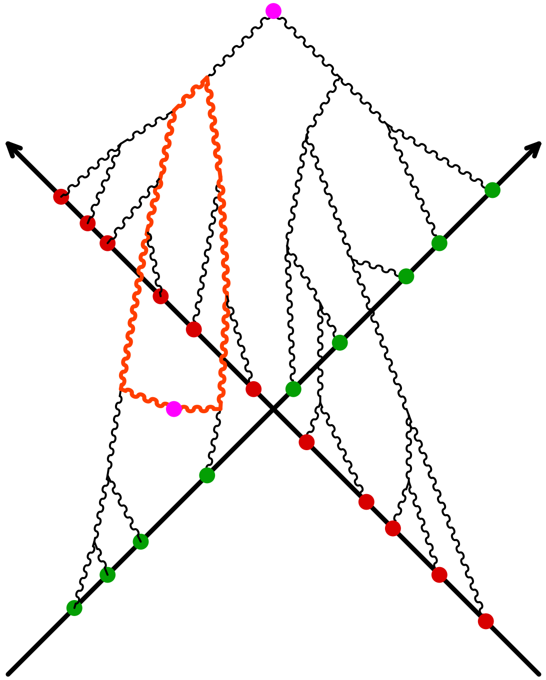
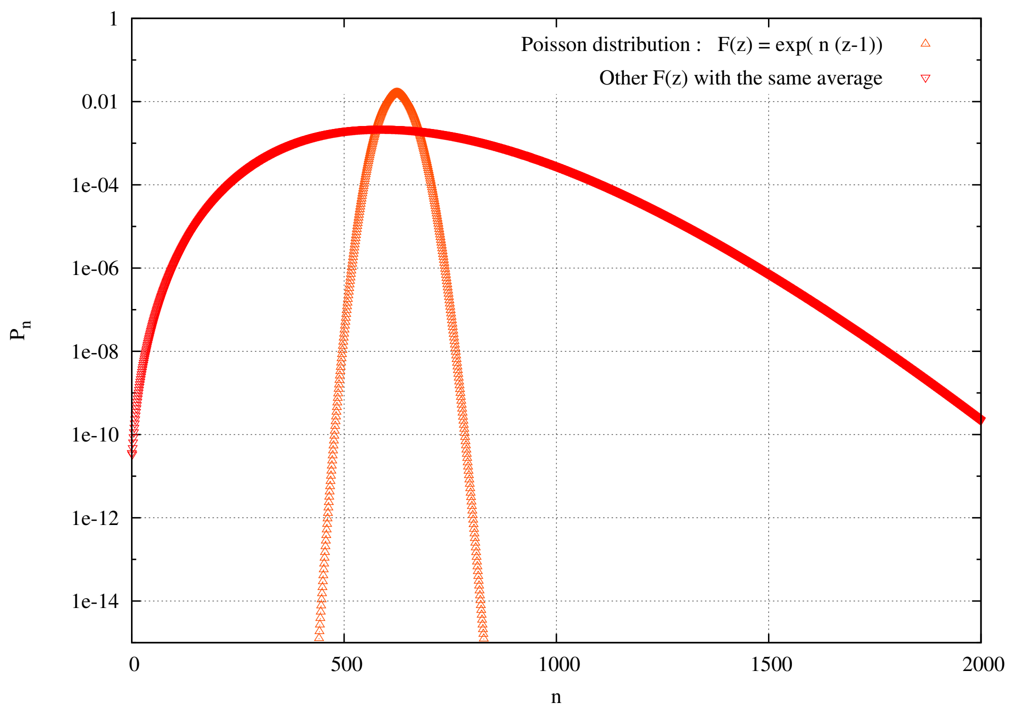


Introduction

- Let us pretend that we know the generating function $F(z)$. We could get the probability distribution as follows :

$$P_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \; e^{-in\theta} \; F(e^{i\theta})$$

Note : this is trivial to evaluate numerically by a FFT :



Derivative of Ln(F(z))

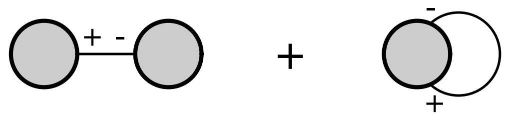
- Reminder :

$$F(z) = \underbrace{e^{z\mathcal{D}[j_+,j_-]}\; e^{iV[j_+]}\; e^{-iV^*[j_-]}}_{e^{iW[z|j_+,j_-]}}\Big|_{j_+=j_-=j}$$

- By the method already used for the multiplicity, we get :

$$\frac{F'(z)}{F(z)} = \frac{1}{Z}\int_{x,y} G^0_{+-}(x,y)\,(\Box_x+m^2)(\Box_y+m^2)\left[\underbrace{\frac{\delta iW}{\delta j_+(x)}\,\frac{\delta iW}{\delta j_-(y)}}+\underbrace{\frac{\delta^2 iW}{\delta j_+(x)\delta j_-(y)}}\right]$$

$$=$$



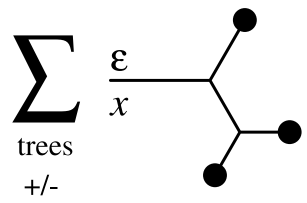
- Note : the topologies involved are the same as in $\langle n\rangle$, but the Feynman rules are modified by :

$$G^0_{+-} \longrightarrow z\,G^0_{+-}\;, \qquad G^0_{-+} \longrightarrow z\,G^0_{-+}$$

Derivative of Ln(F(z))

- At leading order, we need only to calculate two objects, $\phi_+(z|x)$ and $\phi_-(z|y)$, given as the sums of the 1-point connected tree graphs :

$$\phi_\epsilon(z|x) \quad =$$



- Note : these tree diagrams must be calculated with the z-dependent modified Feynman rules

- One can show that these objects are also solutions of the classical equation of motion :

$$(\square_x + m^2)\phi_\pm(z|x) + \frac{g}{2}\,\phi_\pm^2(z|x) = j(x)$$

F(z) from solutions of the EOM

- Write the fields $\phi_\pm(z|x)$ as a superposition of plane waves :

$$\phi_+(z|x) \equiv \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}} \left\{ f_+^{(+)}(x^0,\vec{\boldsymbol{p}})e^{-ip\cdot x} + f_+^{(-)}(x^0,\vec{\boldsymbol{p}})e^{ip\cdot x} \right\}$$

$$\phi_-(z|x) \equiv \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}} \left\{ f_-^{(+)}(x^0,\vec{\boldsymbol{p}})e^{-ip\cdot x} + f_-^{(-)}(x^0,\vec{\boldsymbol{p}})e^{ip\cdot x} \right\}$$

- In terms of these coefficients, the boundary conditions are :

$$f_+^{(+)}(-\infty,\vec{\boldsymbol{p}}) = f_-^{(-)}(-\infty,\vec{\boldsymbol{p}}) = 0$$

$$f_-^{(+)}(+\infty,\vec{\boldsymbol{p}}) = z\, f_+^{(+)}(+\infty,\vec{\boldsymbol{p}})$$

$$f_+^{(-)}(+\infty,\vec{\boldsymbol{p}}) = z\, f_-^{(-)}(+\infty,\vec{\boldsymbol{p}})$$

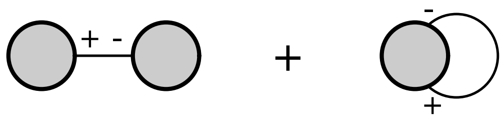
- Finally, at leading order, we obtain :

$$F(z)\big|_{LO} = \exp \int_1^z dz' \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}}\; f_+^{(+)}(+\infty,\vec{\boldsymbol{p}})\, f_-^{(-)}(+\infty,\vec{\boldsymbol{p}})$$

Difficulties of the standard approach

- Reminder :

$$\langle n \rangle =$$



- In principle, by calculating this to all orders, one would obtain the full answer for the number of produced particles

- However, this is complicated by secular divergences :

  ▷ the terms that are dominant at large times are not the same as those that dominate the physics at early times

  ▷ the organization in powers of the coupling is not very relevant, because some higher order corrections may get enhanced by powers of time

- The Dyson-Schwinger equations can resum these secular divergences and make the result sensible. Under certain approximations, they can be simplified into kinetic equations

Alternate approach

- Consider an ensemble of particles on top of the fields :

$$G_{++}(X,p) = \frac{i}{p^2 - m^2 + i\epsilon} + 2\pi f(X,p)\delta(p^2 - m^2)$$

- The distribution f that appears in the propagators is the initial distribution of particles. Of course, by taking $f = 0$ (no particles in the initial state) and by summing all the diagrams, we get the same answer as before (with the same problems related to secular divergences)
- By letting the distribution f evolve in time, we can resum the secular terms
- Strategy :
  - Write the Dyson-Schwinger equations
  - Do a gradient approximation in order to turn them into a Boltzmann equation

Dyson-Schwinger equation

- Because of the external source, the 2-point function has a connected and a disconnected part :

$$\boldsymbol{G}(x,y) \equiv \underbrace{\boldsymbol{G}^{\mathrm{c}}(x,y)}_{\text{connected}} + \underbrace{\boldsymbol{G}^{\mathrm{nc}}(x,y)}_{\text{disconnected}}$$

$$\boldsymbol{G}^{\mathrm{nc}}(x,y) = \big\langle\phi(x)\big\rangle\big\langle\phi(y)\big\rangle$$

- Extract a mean-field term from the self-energy :

$$\boldsymbol{\Sigma}(x,y) \equiv g\boldsymbol{\Phi}(x)\delta(x-y) + \boldsymbol{\Pi}(x,y)$$

- The connected part obeys :

$$\big[\square_x + m^2 + g\boldsymbol{\Phi}(x)\big]\boldsymbol{G}^{\mathrm{c}}(x,y) = -i\delta(x-y) - \int d^4u\;\boldsymbol{\Pi}(x,u)\,\boldsymbol{G}^{\mathrm{c}}(u,y)$$

Dyson-Schwinger equation

- Write the field expectation value in terms of an "effective source" :

$$\langle\phi(x)\rangle \equiv \int d^4u\, \boldsymbol{G}^{\mathrm{c}}(x,u)\, \boldsymbol{S}(u)$$

- The disconnected part obeys :

$$\left[\square_x+m^2+g\boldsymbol{\Phi}(x)\right]\boldsymbol{G}^{\mathrm{nc}}(x,y) = -i\boldsymbol{S}(x)\langle\phi(y)\rangle-\int d^4u\, \boldsymbol{\Pi}(x,u)\, \boldsymbol{G}^{\mathrm{nc}}(u,y)$$

- Then, for the full 2-point function, we get :

$$\left[\square_x + m^2 + g\boldsymbol{\Phi}(x)\right]\boldsymbol{G}(x,y) = -i\delta(x-y) - \int d^4u\, \left[\boldsymbol{\Pi}(x,u)\, \boldsymbol{G}(u,y) + \underbrace{\boldsymbol{\Pi}_S(x,u)}\, \boldsymbol{G}^{\mathrm{c}}(u,y)\right]$$

source term : $-\,i\boldsymbol{\Pi}_S(x,y) \equiv \boldsymbol{S}(x)\,\boldsymbol{S}(y)$

Boltzmann equation

- Quasi-particle ansatz :

$$\boldsymbol{G}_{-+}(X,p) = (1 + f(X,p))\boldsymbol{\rho}(X,p)$$
$$\boldsymbol{G}_{+-}(X,p) = f(X,p)\boldsymbol{\rho}(X,p)$$
$$\boldsymbol{\rho}(X,p) = \boldsymbol{G}_{-+}(X,p) - \boldsymbol{G}_{+-}(X,p)$$

- Wigner transform and gradient expansion :

$$\left[\square_x + m^2 + g\boldsymbol{\Phi}(x)\right]\boldsymbol{G}(x,y) = -i\delta(x-y) - \int d^4u \left[\boldsymbol{\Pi}(x,u)\,\boldsymbol{G}(u,y) + \boldsymbol{\Pi}_S(x,u)\,\boldsymbol{G}^c(u,y)\right]$$

$$2p\cdot\partial_X f(X,p) + g\partial_X\boldsymbol{\Phi}(X)\cdot\partial_p f(X,p) = (1+f(X,p))\boldsymbol{\Pi}_{+-}(X,p) - f(X,p)\boldsymbol{\Pi}_{-+}(X,p) + \boldsymbol{\Pi}_S(X,p)$$

▷ Boltzmann equation with a source term

Summary and perspectives

- In a field theory coupled to strong time-dependent sources, the problem of particle production is non-perturbative, and requires to sum an infinity of diagrams at each order
- Leading Order :
  - The multiplicity can be found from retarded solutions of the classical EOM
  - The generating function requires solutions of the classical EOM with more complicated boundary conditions
- Next to Leading order : one needs the retarded solution of the equation for small field fluctuations in order to calculate the multiplicity
- One can obtain a Boltzmann equation that interpolates between a regime dominated by fields and a regime dominated by particles
- Extensions :
  - Rapidity gaps, diffraction
  - Evolution equation (à la BK) for the generating function

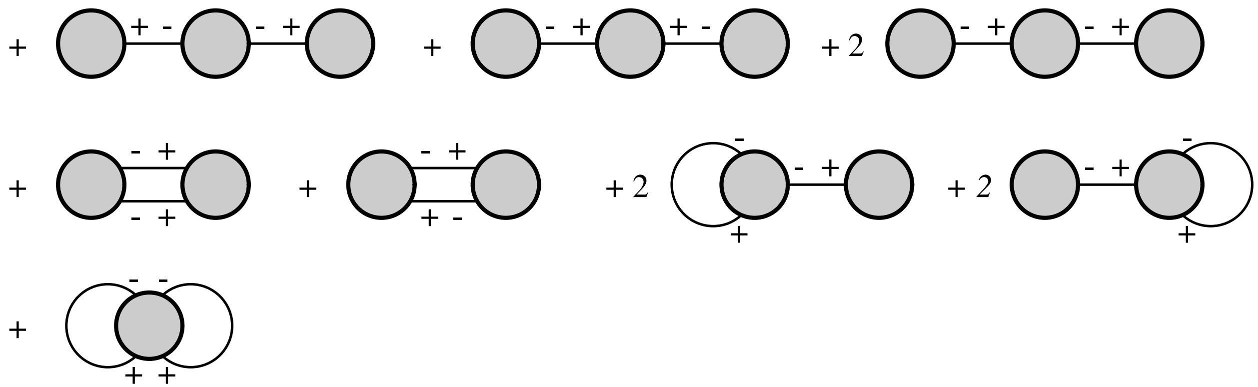
Calculation of the moments

- The same method can be applied to the calculation of the variance :

$$\left\langle n^2 \right\rangle - \left\langle n \right\rangle^2 = \frac{d^2}{dx^2} \ln(F(e^x))_{x=0}$$
$$= \left[\mathcal{D}[j_+, j_-] + \mathcal{D}^2[j_+, j_-]\right] \; e^{iV[j_+, j_-]} \Big|_{\substack{j_+=j_-=j \\ \text{connected}}}$$

- In terms of diagrams :

$$\left\langle n^2 \right\rangle - \left\langle n \right\rangle^2 = \left\langle n \right\rangle$$



Number of independent subdiagrams

- Let us call b_r the sum of all the vacuum-vacuum diagrams in the Schwinger-Keldysh formalism with r cut lines. We have

$$\ln(F(z)) = \sum_{r=1}^{+\infty} b_r\,(z^r - 1)$$

- From this form of the generating function, one gets :

$$P_n = \sum_{p=0}^{n} \underbrace{e^{-\sum_r b_r}\,\frac{1}{p!}\sum_{\alpha_1+\cdots+\alpha_p=n} b_{\alpha_1}\cdots b_{\alpha_n}}_{\text{probability of producing } n \text{ particles in } p \text{ cut subdiagrams}}$$

- Summing on n, we get the probability of p cut subdiagrams :

$$R_p = \frac{1}{p!}\left[\sum_{r=1}^{\infty} b_r\right]^p e^{-\sum_r b_r}$$

Note : this is a Poisson distribution of average $\langle N_{\text{diagrams}}\rangle = \sum_r b_r$

Survival probability

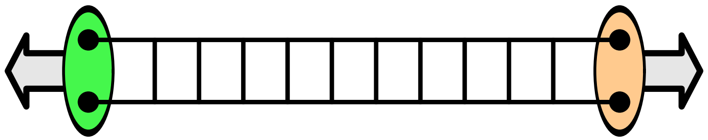
- One interesting quantity to consider is the probability of not producing anything :

$$\begin{aligned} P_0 &= \exp\left(-\sum_r b_r\right) \\ &= \exp\left(-\langle N_{\text{diagrams}}\rangle\right) \end{aligned}$$

- Notes :
  - When defined over a restricted part of phase-space, this quantity is the survival probability for a void in this region of phase-space
  - If the dynamics of the theory allows to have many particles produced in the same subdiagram, it is much larger than the $\exp(-\langle n\rangle)$ one would naively predict from Poisson formula
  - Calculating the survival probability is equivalent to calculating $F(0)$

Rapidity gaps

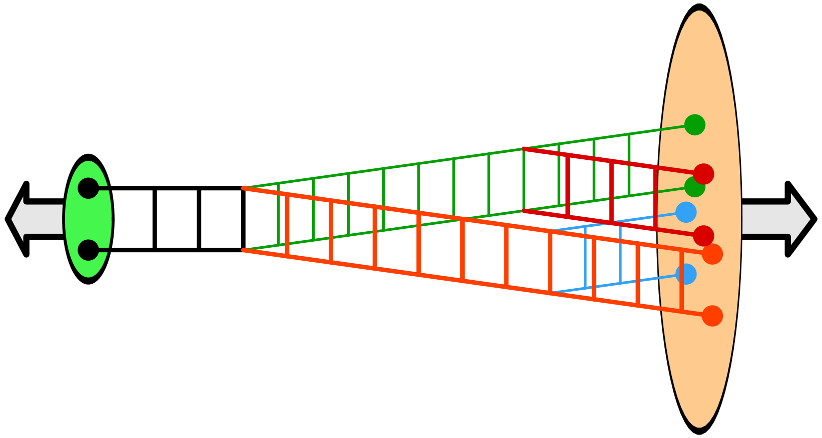
- Proton-proton :



- Thanks to the pomeron, many particles can be produced from the same subdiagram in large rapidity intervals
- Low density of sources $\triangleright$ few pomerons
- The probability of rapidity gaps is not very suppressed, and is largely independent of the gap (position and size)

Rapidity gaps

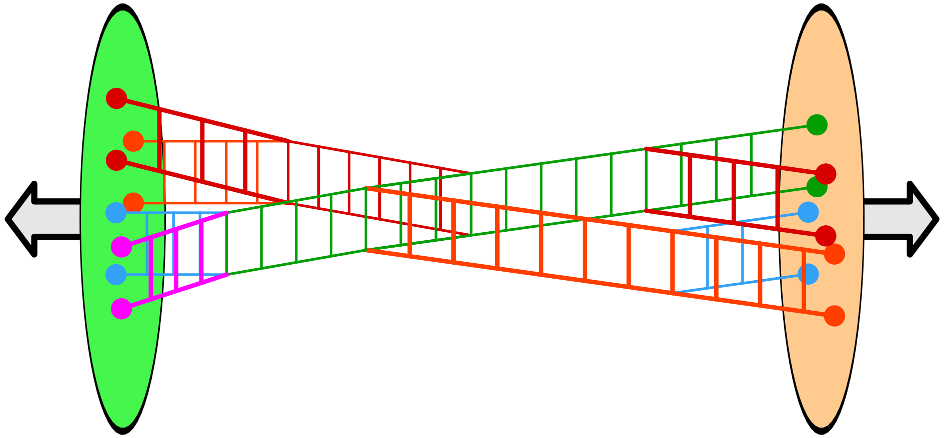
- Proton-nucleus :



- Large density of sources per unit of transverse area in the nucleus ▷ pomeron branchings
- Thanks of these branchings, the number of disconnected subdiagrams does not increase much ▷ rapidity gaps are not much suppressed compared to proton-proton

Rapidity gaps

- Nucleus-nucleus :



- Large density of pomerons
- The probability of rapidity gaps is low
- Diffraction occurs only in peripheral collisions